Iterative equalization for underwater acoustic channels Potentiality for the TRIDENT system.

G. LAPIERRE♣, Joel LABAT♦, Joel TRUBUIL♦

♦ENST Bretagne, Technopôle de Brest Iroise, France

♣GESMA, BP42, 29240 Brest Naval, France

Abstract: **This paper deals with blind adaptive equalization for short burst communication system. Until now, blind adaptive equalizers have been mainly dedicated to continuous data stream applications because of their supposedly well-known slow convergence. The aim of the paper is twofold. It intends both to combat this latter idea and to stress the relevance of the blind approach, under two mild conditions of sufficient burst length and channel stationarity. In some cases, conventional results of equalization are not adequate. This paper proposes two different techniques for improving the performance of equalization processing. For that purpose, the blind decision feedback equalizer introduced in [1] is combined with an iterative procedure and operates on both direct and time reverse sequences. Performances are illustrated with examples of 4-PSK short bursts and underwater acoustic signals. The results are very convincing since, at a low computational cost, the challenge of recovering all the data has now been taken up.**

I- INTRODUCTION

Decision Feedback Equalizer (DFE) is known to be one of the most appropriate structure for the purpose of high data rate acoustic communications both in shallow and deep water [2]. Maximum likelihood sequence estimators (MLSE) cannot always be implemented because of their computational complexity, especially when the delay spread and/or the order of the modulation is high. For those reasons equalizers represent a good trade-off between performance and complexity. In addition, even if channels can often be considered as stationary, they may suffer from an important frequency drift between local oscillators and/or a Doppler effect. That is why, in practice, equalizers need to be adaptive and, for this purpose, the minimum mean squared error (MMSE) criterion is commonly used with the stochastic gradient least mean squares (SGLMS) algorithm. However, even in a trained mode and use of spatial or temporal diversity, performance obtained by conventional equalizer may be not adequate. For example, the time required for the convergence of such adaptive equalizers may be too long to be compatible with burst communication systems. The main goal of this paper is to highlight some techniques based on blind equalization to cope with such situations. At first glance, this solution may seem unrealistic, irrelevant or at least questionable since blind equalization seems mainly dedicated to continuous data flow systems. Nevertheless, despite the importance of the challenge, blind adaptive equalization proves to be a relevant and attractive solution. For this purpose, the blind (single or possibly multiple input) decision feedback equalizer (DFE) described in [1] is particularly well suited. Hereafter, it will be called the SOCMIDFE for self-optimized configuration multiple inputs DFE. Basically, this equalizer is characterized by the existence of two specific running modes. In the acquisition mode, it acts as a linear equalizer while in the tracking mode it acts as the conventional DFE.

This paper is organized as follows. The second part recalls the principle of the blind SOCMIDFE and explains the iterative procedure. The third and fourth part investigate both the principle and the interest of equalizing the time reverse sequence. The fifth part consists of some results given by iterative equalization on synthetic and real signals.

II- RECALLS ABOUT THE SOCMIDFE

Let d_n denote a zero-mean, unit power, independent and identically distributed (i.i.d.) sequence of discrete data transmitted through P channel $\mathcal{H}p\ (p \in [1:P])$, with a transfer function (TF) $H_p\{z\}$ of order N_p. Let N be the greatest value.

$$N = \max_{p\in[1:P]}\left(N_p\right) \tag{1}$$

Each signal is corrupted by an additive white Gaussian noise $n_p(k)$, with variance $\sigma^2_{n_p}$, in such a way that the observed signal $s_p(n)$ can be written as :

$$s_p(k) = \sum_{l=0}^{N} d_{k-l} h_p(l) + n_p(k) \tag{2}$$

For what follows, each variance of additive noise will be the same, denoted σ_n^2 and noise received by two different sensors are not correlated. We recall that the linear MMSE equalizer (Wiener solution), has the following TF :

$$C_p\{z\} = \frac{\sigma_d^2 H_p^*\left\{1/z^*\right\}}{\sigma_d^2 \sum_{p=1}^{P} H_p^*\left\{1/z^*\right\} H_p\{z\} + \sigma_n^2} \tag{3}$$

Usually, this multiple input linear equalizer (LE) is implemented as a transversal (FIR) filter in order to avoid unstabilities. However, a recursive (IIR) implementation, as described in [1], can bring substantial advantages since :

1- it generally requires less coefficients than its purely transversal (FIR) counterpart.

2- the optimal order of the feedback filter is known to be *N*.

3- it allows the linear equalizer to easily evolve towards the conventional DFE.

In fact, the denominator of $C_p\{z\}$, $D\{z\}$ has $2N$ roots, N being inside the unit circle and N outside.
Hence, $D\{z\}$ can be factorized as :

$$D\{z\} = A_p G\{z\} G^*\{1/z^*\} \tag{4}$$

where $G\{z\}$ is the minimum phase polynomial and A_P a positive (real) constant :

$$G\{z\} = \prod_{i=1}^{N}\left(1 - z_i z^{-1}\right) \text{with } |z_i| < 1 \tag{5}$$

As a result, the equalizer TF $C_p\{z\}$ can be then written as :

$$C_p\{z\} = \frac{1}{G\{z\}} \frac{\sigma_d^2}{A_P} \frac{H_p^*\{1/z^*\}}{G^*\{1/z^*\}}. \tag{6}$$

It clearly appears that the filter with TF $1/G\{z\}$ can be implemented as a stable purely recursive filter. So, the optimal linear equalizer can be seen as the cascade of a feedback filters (FBF) with TF $1/G\{z\}$ and P feedforward filters (FFF) with TF $\frac{\sigma_d^2}{A_p} \frac{H_p^*\{1/z^*\}}{G^*\{1/z^*\}}$. Hence, from a structural point of view, the optimal linear equalizer can be implemented as depicted in Fig. 1.

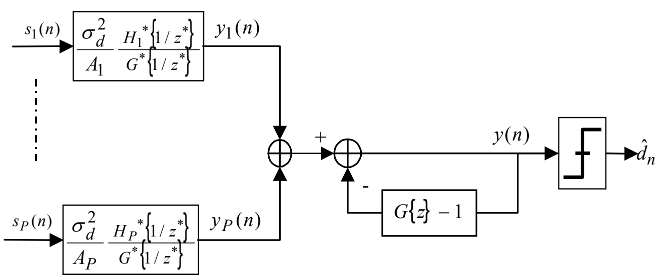


Fig. 1: Optimal linear equalizer (acquisition mode).

Naturally since this equalizer is linear, the order of the two stages (FBF and FFF) is irrelevant from a structural point of view. However, in an adaptive blind approach, the FBF must be placed before FFF. That is why this filter is duplicated on each branch. On the other hand, the conventional DFE has the structure depicted in Fig. 2. So, the main difference between the MMSE-LE and the conventional MMSE-DFE is that the FBF is now fed in with detected data . So, at the (little) price of a structural modification, one can switch from a linear equalizer to a DFE, and conversely, in a straightforward manner. In the blind approach, the remaining problem is to find criteria leading to a solution closely related to the MMSE solution. Every detail can be found in [1] where algorithms for adaptation are detailed.
Basically, the novel equalizer [1] is characterized by the existence of two specific running modes. In the first one, termed the acquisition mode, the equalizer is linear, recursive and "blindly" adapted according to relevant criteria. The FBF, named A is adapted by minimizing the cumulated output power while the FFFs, named B_p are adapted according to the Godard criterion [3]. Once DD-MSE falls under a given threshold, it is then driven in the tracking mode, where it acts as the standard decision-directed DFE (see Fig. 2). Relevant signals such as the Godard error [3] or other cost functions [4], [5] and [6] can be estimated in order to select the appropriate running mode.

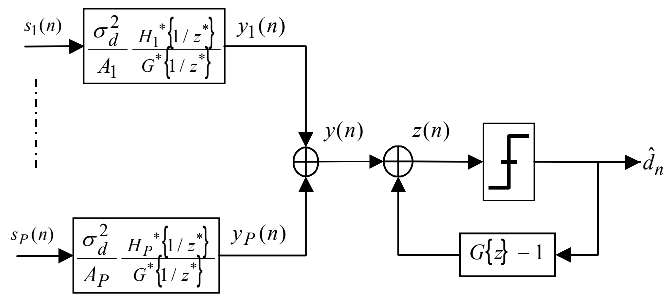


Fig. 2: Conventional DFE (tracking mode).

To sum up, here are recalled the main features of the SOCMIDFE.

Acquisition mode

1- Criteria are specific for each stage, that is, local instead of global. The FBF is adapted to minimize the cumulated output powers while the FFFs are adapted to satisfy the Godard criterion.

2- For the FBF adaptation, the criterion leads to a solution closely related to the MMSE solution [1].

3- The FFFs (resp. FBF) transfer function are the same for both the LE and the DFE, allowing to simply switch from linear (LE) to non linear (DFE) and conversely. The only difference is given by the nature of the input signals of FBF filter A (Fig. 3).

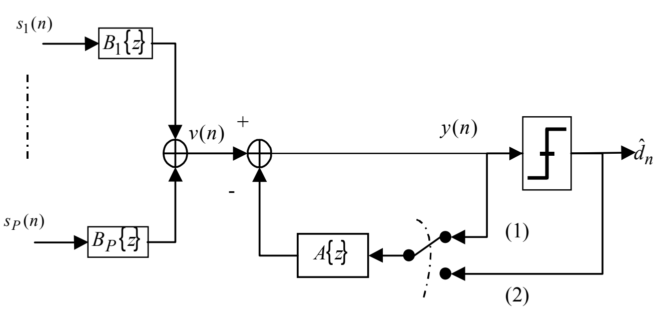


Fig. 3 : Switching principle.

Tracking mode

1- When coefficients are close enough to their final values, the switching occurs.

2- Whenever the equalizer is lost because of the channel severity, which is detected by observing an appropriate signal, the equalizer is then switched back to the acquisition mode and so on.

Naturally, despite its very high speed of convergence [7], such an equalizer is not fast enough to be used, as it is, in burst communication systems. This is a well-known drawback of blind (and even trained) adaptive equalizers. However, provided that an iterative procedure is performed, the time required for convergence can be significantly reduced and even virtually cancelled.

Iterative procedure. Basically, this strategy relies on the fact that, at the end of an iteration, the estimated equalizer parameters are expected to be closer to their optimal values than at the very beginning of the same iteration. As a consequence, initializing the equalizer parameters of a given iteration, by those estimated at the end of the previous one, allows the SOCMIDFE to converge on short bursts, which makes its attractive and fully compatible with most of today's applications. This procedure, which is completely meaningless in a trained approach, turns out to be particularly relevant and efficient in a blind strategy, at a quite reasonable computational burden. Basically this is because, in a blind approach, every sample of the burst equally contributes to the equalization process while, in a trained approach, only the few samples carrying the known sequence are reliable enough to be used in such a procedure. However, it is worth recalling that two conditions need to be fulfilled : namely sufficient burst length (1000 symbols for 4-PSK) and channel stationarity, which is not too constraining in practice.

In order to still improve the global performance of the SOCMIDFE, equalization can also be simultaneously processed on the time reversed sequence. The next section recalls both the principle and its application.

III- TIME REVERSE EQUALIZATION

Let $C_p\{z\}$ denote the linear equalizer TF and $S_p\{z\}$ the *z*--transform of $s_p(k)$, then the *z*-transform of the time reversed sequence $s_p^*(-k)$ is $S_p^*\{1/z^*\}$, in such a way that the linear filter TF is :

$$C_p^*\{1/z^*\} = \frac{1}{G\{z\}}\frac{\sigma_d^2}{A_P}\frac{H_p\{z\}}{G^*\{1/z^*\}} \quad (7)$$

Once again, this new TF can be split into the cascade of a whitening FBF, with TF $1/G\{z\}$ and a FFF, with TF $\frac{\sigma_d^2}{A_p}\frac{H_p\{z\}}{G^*\{1/z^*\}}$. Obviously, the SOCMIDFE [1] can be used as well. In addition, one can note that the FBF is the same in both cases. Moreover, although the performance is identical in terms of MMSE, it may substantially differ in terms of speed of convergence.

The first interest of this method is that, after a time reverse procedure, a maximum phase channel is now seen as minimum phase and conversely. This feature allows to substantially improve the speed of convergence in a given sense of equalization.

For instance, given a minimum phase channel TF :

$$H\{z\} = 1 - z_0 z^{-1} \text{ with } |z_0| < 1 \quad (8)$$

and according to (1), one can write :

$$S\{z\} = D\{z\}H\{z\} + N\{z\} \quad (9)$$

and therefore it comes :

$$S^*\{1/z^*\} = D^*\{1/z^*\}H^*\{1/z^*\} + N^*\{1/z^*\} \quad (10)$$

with

$$H^*\{1/z^*\} = 1 - z_0^* z \quad (11)$$

The sequences $d^*(-k)$ (resp. $n^*(-k)$) have the same statistical properties as the sequences $d(k)$ (resp. $n(k)$). As a consequence, the new channel TF $H^*\{1/z^*\}$ has its zero outside the unit circle, since $\left|1/z_0^*\right| > 1$. Otherwise stated, the new channel is maximum phase, i.e., much more difficult to equalize. At the opposite, when the channel $H\{z\}$ is maximum phase, $H^*\{1/z^*\}$ becomes minimum phase, and so much easier to equalize. Therefore one can expect to seriously increase the speed of convergence by processing both direct and reversed sequence equalization and selecting the best sense, for a given channel. This mode is called select mode. Numerical simulations will corroborate this result.

In the noiseless case, given the previous minimum phase channel, one can write, in a single input single output (SISO) context :

$$C_1\{z\} = \frac{1}{H\{z\}} = \frac{1}{1 - z_0 z^{-1}} \quad (12)$$

So, by identifying (7) (for $p = 1$) and (12) leads to :

$$G\{z\} = 1 - z_0 z^{-1} \text{ and } \frac{\sigma_d^2}{A_1}\frac{H_1\{z\}}{G^*\{1/z^*\}} = 1 \quad (13)$$

Roughly speaking, this mean that the transversal and recursive filters just need one non zero tap. On the contrary,

$$\frac{1}{H^*\left\{1/z^*\right\}} = \frac{1}{1 - z_0 z^{-1}} \frac{1 - z_0 z^{-1}}{1 - z_0^* z} \quad (14)$$

that is (15):

$$\frac{1}{H^*\left\{1/z^*\right\}} = \frac{1}{1 - z_0 z^{-1}} \left[-z_0 z^{-1} + \left(1 - |z_0|^2\right) \sum_{k=0}^{+\infty} \left(z_0^* z\right)^k \right]$$

which means that the recursive filter has the same TF as in (14), while the transverse filter is now an all-pass filter which needs several taps (instead of one in the previous case). That is the reason why, in an adaptive approach, the time of convergence can be significantly reduced when processing equalization in the appropriate sense. This choice can be selected *a posteriori* on the basis of a relevant signal such as the DD-MSE or Godard function, for example.

Naturally, in the more general case, communication channels are neither minimum nor maximum phase since their TFs have zeros inside and outside the unit circle. For that reason this procedure does not always bring such a great improvement in every case. Nevertheless, in combining the two approaches, namely the iterative procedure and time reverse equalization, one can expect to substantially improve the global performance.

2 - The second great advantage provided by this method is that, in the case of conventional equalization, the detected data have a poor reliability at the beginning of the sequence, because of the time required for convergence. On the other hand, when a time reversal operation is carried out, errors still affect the beginning of the sequence $w^*(-k)$, that is, the end (instead of the beginning) of the sequence $w(k)$. This means that performing both direct and time reverse equalization allows to recover the whole transmitted data. So cleverly combining both strategies can lead to a significant improvement. This mode is called diversity combining mode. Finally, everything happens as if the time required for convergence was virtually cancelled or at least drastically reduced, which is a very spectacular result.

IV- DIVERSITY COMBINING

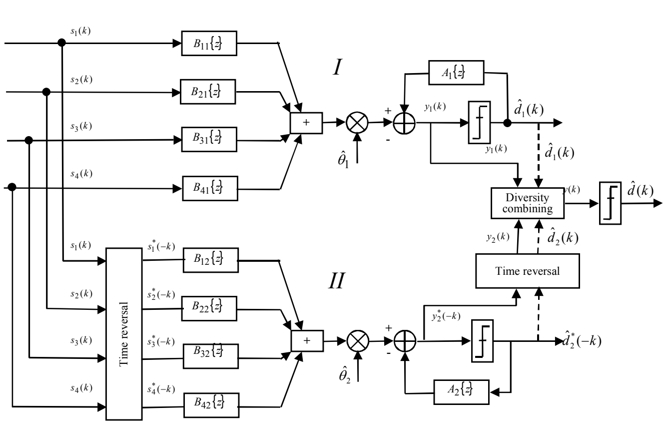


Fig. 4: Time reversal SOCMIDFE structure, BISOCMIDFE.

The receiver structure of
Fig. 4 processes data using two different streams. The 4-sensors received signals are equalized in stream I using the SOCMIDFE presented on previous part. Received signals are time-reversed in stream II. After that, they are equalized with the SOCMIDFE structure. Resulted data are time-reversed once again. The soft output $y(n)$ of the diversity combining block is given by a simple linear combination of the sequences $y_1(n)$ and $y_2(n)$.

$$y(n) = \alpha y_1(n) + (1-\alpha) y_2(n) \quad (15)$$

The soft input sequences $y_1(n)$ and $y_2(n)$ can be expressed as,

$$y_1(n) = s_1(n) + e_1(n) \quad (16)$$

$$y_2(n) = s_2(n) + e_2(n) \quad (17)$$

where $e_1(n)$ and $e_2(n)$ stand for the decision error on direct and time-reversed equalization. The MSE for the soft output $y(n)$ is given by (17):

$$MSE = E\left\{\left|w(n) - d_n\right|^2\right\} \quad (18)$$

$$MSE = E\left\{\left|\alpha e_1[n] + (1-\alpha) e_2[n]\right|^2\right\} \quad (19)$$

$$MSE = \alpha^2 E\left\{\left|e_1(n)\right|^2\right\} + (1-\alpha)^2 E\left\{\left|e_2(n)\right|^2\right\} + 2\alpha(1-\alpha) \underbrace{Re\left\{E\left\{e_1(n) e_2^*(n)\right\}\right\}}_{\rho_n} \quad (20)$$

The weighting factor is optimized to minimize MSE and the optimal value is obtained by setting $\frac{\partial MSE}{\partial \alpha} = 0$,

$$\frac{\partial MSE}{\partial \alpha} = \alpha\left[MSE_1(n) + MSE_2(n)\right] + (1-2\alpha)\rho_n - MSE_2(n) = 0 \quad (21)$$

it comes ,

$$\alpha(n) = \frac{MSE_2(n) - \rho_n}{MSE_1(n) + MSE_2(n) - 2\rho_n} \quad (22)$$

When $MSE_1(n) = MSE_2(n)$, the optimal weighting factor is $\alpha = 1/2$. It is the equal gain combining.

One of the main difficulties concerning the use of the BISOCMIDFE structure is the data synchronization. In blind approach, inherent detection delay given by direct and inverse mode could be different. In case of harsh channel for one of the two streams, correlation between decided data on stream I and II could be hard to achieve. The diversity combining method could be in that case very problematic. In [8], simulation results present better performances for diversity combining mode rather than select mode. These results are obtained with a supervised approach (the decision that are fed back to the FBF of the DFE are correct).

V- NUMERICAL RESULTS

A. QPSK signaling.

In what follows, we investigate QPSK signals transmitted in burst format in a single input single output (SISO) context, each burst consisting of 1050 symbols including a prefix and an end of file of 25 symbols each. One thousand channels have been randomly generated, each of them having 4 zeros. In addition, for each channel, ten files have been investigated. The bit error rate (BER) was evaluated on the basis of the 1000 useful symbols, with a signal to noise ratio (*snr*) ranging between 10 dB and 15 dB. The number of taps for the SOCMIDFE, is 21 in the transverse filter $\mathbf{B}$ and $N = 5$ in the recursive filter. The initial tap coefficient vectors are $\mathbf{A}(0) = [0,0,0,0]^T$ and $\mathbf{B}(0) = [0,...,0,0.5,0,...,0]^T$. Only the 18^{th} tap is non-zero. The step size used for adaptation of the filters is 0.003 and 0.01 for the AGC. Results are expressed in terms of bit error rate (Fig. 5).

The bit error rate was evaluated for the last four described strategies. The first one consists in conventional equalization operating on the observed sequence, associated with an iterative procedure (10 iterations). In what follows, it will be termed as direct mode. The second one consists in equalizing the time reversed sequence, it will be termed as reversed mode. The third one consists in selecting the best realization to evaluate the BER, it will be termed as select mode. Finally, the last one consists in combining decided data given by the first two modes. It will be called diversity combining mode.

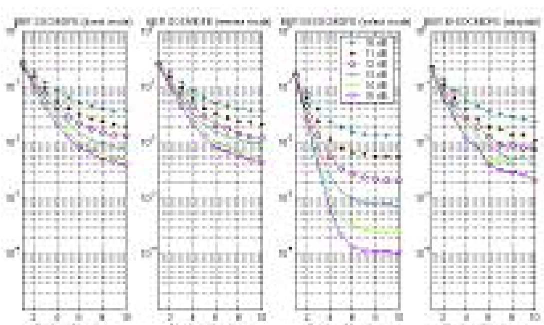


Fig. 5: BER : direct equalization (left), time reversed equalization (left middle), select mode equalization (right middle) and diversity combining equalization (right).

Roughly speaking, it appears that direct and reversed equalizations approximately lead to similar results, while the third strategy brings a very important gap in terms of BER. This result is due to the fact that, in most cases, equalization is seldom difficult on both direct and reversed sequences. These results mean that the **blind** SOCMIDFE typically requires less than 25 symbol periods for convergence, since the following data are totally recovered, which is an outstanding result fully comparable to an adaptive **trained** procedure involving a recursive least squared algorithm. Diversity combining results do not exhibit any performance gain. Nevertheless, the same simulation realized with supervised equalization confirm conclusions of [8]. This could be explained by the difficulty to synchronize the two decided data obtained with stream I and II when blind equalization is considered.

B- UWA signals

The second example corresponds to the transmission of a Q-PSK modulated signal with a bit rate of 10 kbps in a surf-zone near Brest harbor. This signal is acquired with the TRIDENT equipment presented during previous sessions of OCEANS [9],[10]. The kind of information is MLBS[1] without coding and scrambling. This type of information is useful to estimate the bit error rate (BER). The distance between transmitter and receiver is about 500 m and the depth around 20m. This short range was chosen to meet a harsh multipath structure.

The plots in Fig. 6 display the evolution of the different impulse responses. The multipath structure exhibit five main paths, the first two of which are the most energizing. Acoustic arrivals are stable except for the second path. Time spread is greater than 100 symbols duration (20ms). This explain the great amount of taps used for recursive filter. The length of the recursive filter is representative of the time spread of impulse response as mentioned in part II-.

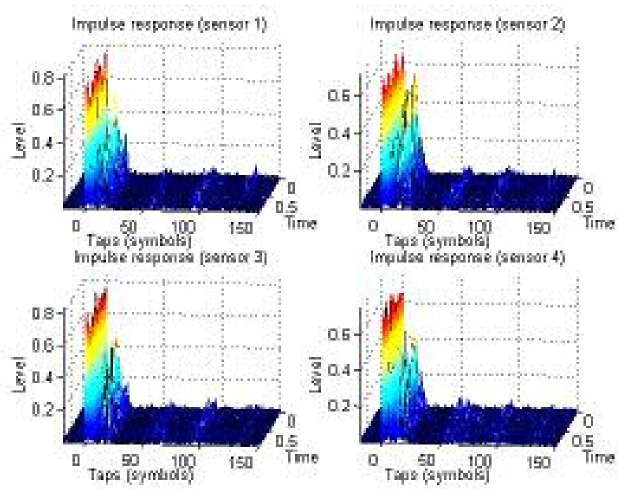


Fig. 6: Evolution of the magnitude of impulse responses during transmission.

Four processing (all based on iterative equalization) are tested on this frame The first one consists in conventional equalization operating on the observed sequence. In what follows, it will be termed as direct mode. The second one consists in equalizing the time reversed sequence, it will be termed as inverse mode. The third one consists in selecting

[1] Maximum Length Binary Sequence

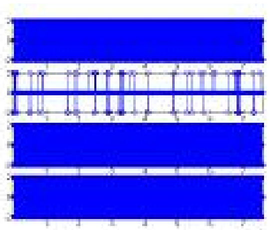
the best realization to evaluate the BER, it will be termed as select mode. The last one consists in combining the decided data given by direct and inverse mode (combined mode). The different parameters of the SOCMIDFE equalizer are the following: 17 taps (the first five of which for the causal part) for each transversal filter and 100 taps for the recursive filter.

40 000 symbols were considered. This frame is divided into short bursts of 2000 symbols length. Iterative equalization is realized on each of these bursts separately. 5 iterations are processed. Sharing continuous streams into short bursts may bring two advantages. First of all, the time required for convergence on each burst may be reduced drastically. Secondly, acoustic channel virtually appears more stable on the duration of each burst. As a consequence, rapid variations of UWA channel may be apprehended. At the end, continuous stream of decided data is rebuilt.

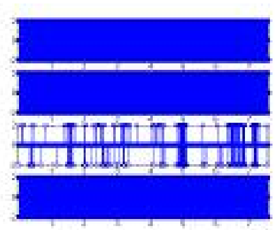
Fig. 7 shows the binary errors distribution resulting from the different processing. Data are correctly estimated up to a phase ambiguity of multiple $\pi/2$ (except for select mode). This is a well-known phenomenon due to the rotational invariance of input signals statistics. On each of these four lines, stems stand for the difference between decided and a rotation of transmitted data.

As a result, the next table provides the error and BER results. In case of successfully correlation between decided data given by direct and inverse mode, the combined mode can exhibit better performances than the select mode ones. Left errors are well-spread during transmission. Channel coding could complete this processing.

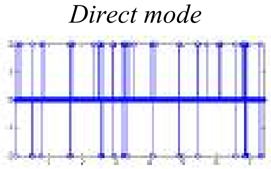
Mode	Length	Number of errors	BER ($.10^{-3}$)
Direct	75739	335	4.4
Inverse	75739	364	4.8
Select	74000	117	1.6
Combined	75739	37	0.5



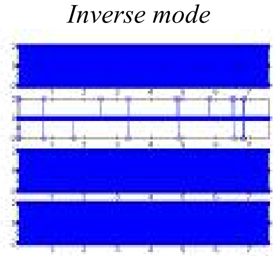
Direct mode



Inverse mode



Select mode



Combined mode

Fig. 7: Evolution of the distribution of binary errors during transmission.

On Fig. 8, one can see the evolution of MSE during transmission. The blue line draws the MSE given by conventional equalization. MSE resulting from inverse equalization is drawn in red. Finally, the yellow line depicts the evolution of MSE with the combined mode.

On Fig. 9, the evolution of the weighting α of direct equalization clearly shows that direct mode and inverse mode are weighted approximately with the same factor (1/2). This could be explained by the fact that some good results are already given by direct and inverse mode. It is the equal-gain combining. Nevertheless, one can see at the beginning of the sequence that inverse mode (weak α) is preferred because of the good reliability at the beginning of the sequence when time-reversed equalization is considered.

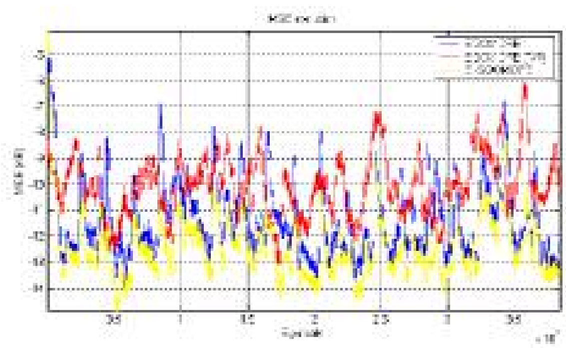


Fig. 8: Evolution of the MSE.

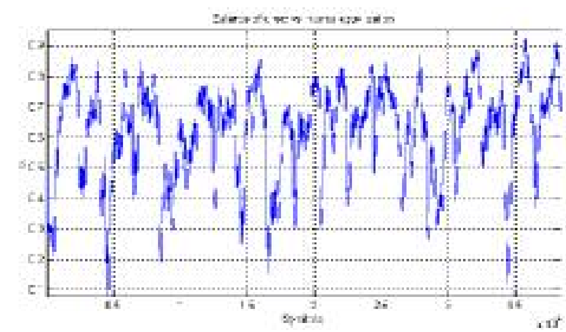


Fig. 9: Evolution of the weighting of direct and inverse mode.

VI- CONCLUSION

Until now, the main drawback of blind adaptive equalizers has been their weak speed of convergence. That is why they have been commonly dedicated to continuous data flow systems. This paper introduces a new concept combining the novel blind adaptive equalizer, termed the SOCMIDFE [1], with an iterative strategy in order to estimate the equalizer parameters. Basically, this approach consists in initializing the equalizer parameters, at iteration of order *n,* by the parameters estimated at the end of the previous iteration and so on. Such a strategy used alone or, still better, in conjunction with a time reversed operation allows the equalizer to deal with (short) data bursts. According to numerical results involving synthetic and real signals, the most important conclusion to be drawn is that this strategy brings promising results, in terms of BER. Roughly speaking, thanks to the iterative procedure, the blind SOCMIDFE typically requires less than 1 symbol duration per coefficient – that is, 25 symbol duration in our example - to converge, which is an outstanding result. On the other hand, when both direct and time reverse equalization are used, the whole data are recovered even at the very beginning of the burst.

REFERENCES:

[1] J. Labat and C. Laot, 'Blind adaptive multiple input decision feedback equalizer with a self optimized configuration', *IEEE trans on Comm*, Vol. 49, N°4, April 2001.

[2] D. Kilfoyle, A. Baggeroer, 'The state of the art in underwater acousuctic telemetry', *IEEE Journal of Oceanic Engineering*, Vol.25, n°1, January 2000.

[3] D. N. Godard, "Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication System", *IEEE Trans. on Comm.*, vol. 28, pp. 1867-1875, 1980.

[4] 0. Shalvi & E. Weinstein, "New Criteria for Blind Deconvolution of NonMinimum Phase Systems (Channels)", *IEEE Trans. on IT*, vol. 36, No 2, March 1990, pp. 312-321.

[5] V Shtrom & H. Fan, "New Class of Zero-Forcing Cost Functions in Blind Equalization", *IEEE Trans. on SP*, vol. 46, No 10, October 1998, pp. 2674-2683.

[6] S. Vembu, S. Verdu, R. A. Kennedy and W. Sethares, "Convex cost function in blind equalization, *IEEE Trans. on signal processing*, vol. 42, pp. 1952-1959, Aug. 94.

[7] J. Labat, O. Macchi and C. Laot,"Adaptive Decision Feedback Equalization : Can you Skip the Training Period ? ", *IEEE Trans. on. Com*., Vol. 46, No 7, pp. 921-930, July 98.

[8] J. Balakrishnan, C. Johnson, 'Time-reversal diversity in decision feedback equalization', in *Proc. of Allerton Conf. On Comm. Control and Computing*, Monticello, IL, USA, Oct. 2000

[9] J Trubuil, G Lapierre, T Gall, J Labat , ' Real-time high data rate acoustic link based on spatio temporal blind equalization: the TRIDENT acoustic system, *Proc. OCEANS 2002*, Biloxi, Vol.4, pp2438-2443.

[10] J Trubuil, T Gall, G Lapierre, J Labat, ' Development of a real-time high dated rate acoustic link ', *Proc. OCEANS 2001*, Hawaii, Vol 4, pp 2159-2164.